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Resumen de Semántica, (I)

2 Sintaxis de IMP

$m, n \in$	**N**	$= \{\ldots,$ `-2, -1, 0, 1, 2` $,\ldots\}$	números enteros
$t \in$	**T**	$= \{$`true, false`$\}$	booleanos
$X, Y \in$	**Loc**	$= \{$`X, Y, Z,` $\ldots\}$	variables
$a \in$	**Aexp**	$=$	expresiones aritméticas
$b \in$	**Bexp**	$=$	expresiones booleanas
$c \in$	**Com**	$=$	comandos

Las clases **Aexp**, **Bexp** y **Com** están especificadas por las siguientes ecuaciones BNF:

a	$::=$	n	b	$::=$	`true`	c	$::=$	`skip`
	|	X		|	`false`		|	`X:=a`
	|	$a_0 + a_1$		|	$a_0 = a_1$		|	$c_0; c_1$
	|	$a_0 - a_1$		|	$a_0 < a_1$		|	`if` b `then` c_0 `else` c_1
	|	$a_0 \times a_1$		|	$\neg b$		|	`while` b `do` c
				|	$b_0 \wedge b_1$			
				|	$b_0 \vee b_1$			

- Un estado: σ : **Loc** $\rightarrow$ **N**. El conjunto de estados :Σ.
- Regla:

$$(\sigma[m/X])\,(X) = m$$

$$(\sigma[m/X])\,(Y) = \sigma(Y) \qquad \text{si } X \neq Y$$

3 Semántica operacional

3.1 $< a, \sigma > \rightarrow n$

$$\frac{}{< n, \sigma > \rightarrow n}$$

$$\frac{}{< X, \sigma > \rightarrow \sigma(X)}$$

$$\frac{< a_0, \sigma > \rightarrow n_0 \quad < a_1, \sigma > \rightarrow n_1}{< a_0 + a_1, \sigma > \rightarrow n} \text{ si } n = n_0 + n_1$$

$$\frac{< a_0, \sigma > \rightarrow n_0 \quad < a_1, \sigma > \rightarrow n_1}{< a_0 - a_1, \sigma > \rightarrow n} \text{ si } n = n_0 - n_1$$

$$\frac{< a_0, \sigma > \rightarrow n_0 \quad < a_1, \sigma > \rightarrow n_1}{< a_0 \times a_1, \sigma > \rightarrow n} \text{ si } n = n_0 \times n_1$$

3.2 $< b, \sigma > \to t$

$$\frac{}{< \texttt{true}, \sigma > \to \texttt{true}}, \quad \frac{}{< \texttt{false}, \sigma > \to \texttt{false}}$$

$$\frac{< a_0, \sigma > \to n \quad < a_1, \sigma > \to m}{< a_0 = a_1, \sigma > \to \texttt{true}} \text{si } n = m, \quad \frac{< a_0, \sigma > \to n \quad < a_1, \sigma > \to m}{< a_0 = a_1, \sigma > \to \texttt{false}} \text{si } n \neq m$$

$$\frac{< a_0, \sigma > \to n \quad < a_1, \sigma > \to m}{< a_0 \leq a_1, \sigma > \to \texttt{true}} \text{si } n \leq m, \quad \frac{< a_0, \sigma > \to n \quad < a_1, \sigma > \to m}{< a_0 \leq a_1, \sigma > \to \texttt{false}} \text{si } n \not\leq m$$

$$\frac{< b, \sigma > \to \texttt{true}}{< \neg b, \sigma > \to \texttt{false}}, \quad \frac{< b, \sigma > \to \texttt{false}}{< \neg b, \sigma > \to \texttt{true}}$$

$$\frac{< b_0, \sigma > \to t_0 \quad < b_1, \sigma > \to t_1}{< b_0 \wedge b_1, \sigma > \to t} t = t_0 \wedge t_1, \quad \frac{< b_0, \sigma > \to t_0, \quad < b_1, \sigma > \to t_1}{< b_0 \vee b_1, \sigma > \to t} t = t_0 \vee t_1$$

3.3 $< c, \sigma > \to \sigma'$

$$\frac{}{< \texttt{skip}, \sigma > \to \sigma} \quad \frac{< a, \sigma > \to m}{< X := a, \sigma > \to \sigma[m/X]}$$

$$\frac{< c_0, \sigma > \to \sigma'' \quad < c_1, \sigma'' > \to \sigma'}{< c_0; c_1, \sigma > \to \sigma'}$$

$$\frac{< b, \sigma > \to \texttt{true} \quad < c_0, \sigma > \to \sigma'}{< \texttt{if } b \texttt{ then } c_0 \texttt{ else } c_1, \sigma > \to \sigma'} \quad \frac{< b, \sigma > \to \texttt{false} \quad < c_1, \sigma > \to \sigma'}{< \texttt{if } b \texttt{ then } c_0 \texttt{ else } c_1, \sigma > \to \sigma'}$$

$$\frac{< b, \sigma > \to \texttt{false}}{< \texttt{while } b \texttt{ do } c, \sigma > \to \sigma}$$

$$\frac{< b, \sigma > \to \texttt{true} \quad < c, \sigma > \to \sigma'' \quad < \texttt{while } b \texttt{ do } c, \sigma'' > \to \sigma'}{< \texttt{while } b \texttt{ do } c, \sigma > \to \sigma'}$$

4 Semántica denotacional

4.1

$\mathcal{A}[\![a]\!]: \Sigma \to \mathbf{N}$

$$\mathcal{A}[\![n]\!]\sigma = n, \quad \mathcal{A}[\![X]\!]\sigma = \sigma(X), \quad \mathcal{A}[\![a_0 + a_1]\!]\sigma = \mathcal{A}[\![a_0]\!]\sigma + \mathcal{A}[\![a_1]\!]\sigma$$

$$\mathcal{A}[\![a_0 - a_1]\!]\sigma = \mathcal{A}[\![a_0]\!]\sigma - \mathcal{A}[\![a_0]\!]\sigma, \quad \mathcal{A}[\![a_0 \times a_1]\!]\sigma = \mathcal{A}[\![a_0]\!]\sigma \times \mathcal{A}[\![a_0]\!]\sigma$$

4.2

$\underline{\mathcal{B}[\![b]\!] \colon \Sigma \to \mathbf{T}}$

$$\mathcal{B}[\![\texttt{true}]\!]\sigma = \texttt{true}, \quad \mathcal{B}[\![\texttt{false}]\!]\sigma = \texttt{false}, \quad \mathcal{B}[\![\neg b]\!]\sigma = \neg(\mathcal{B}[\![b]\!]\sigma)$$

$$\mathcal{B}[\![a_0 = a_1]\!]\sigma = \begin{cases} \texttt{true} & \text{si } \mathcal{A}[\![a_0]\!]\sigma = \mathcal{A}[\![a_1]\!]\sigma \\ \texttt{false} & \text{si } \mathcal{A}[\![a_0]\!]\sigma \neq \mathcal{A}[\![a_1]\!]\sigma \end{cases}$$

$$\mathcal{B}[\![a_0 \leq a_1]\!]\sigma = \begin{cases} \texttt{true} & \text{si } \mathcal{A}[\![a_0]\!]\sigma \leq \mathcal{A}[\![a_1]\!]\sigma \\ \texttt{false} & \text{si } \mathcal{A}[\![a_0]\!]\sigma \not\leq \mathcal{A}[\![a_1]\!]\sigma \end{cases}$$

$$\mathcal{B}[\![b_0 \wedge b_1]\!]\sigma = (\mathcal{B}[\![b_0]\!]\sigma) \wedge (\mathcal{B}[\![b_1]\!]\sigma)$$

$$\mathcal{B}[\![b_0 \vee b_1]\!]\sigma = (\mathcal{B}[\![b_0]\!]\sigma) \vee (\mathcal{B}[\![b_1]\!]\sigma)$$

4.3

$\underline{\mathcal{C}[\![c]\!] : \Sigma \rightharpoonup \Sigma}$

$$\mathcal{C}[\![\texttt{skip}]\!]\sigma = \sigma, \quad \mathcal{C}[\![X := a]\!]\sigma = \sigma[(\mathcal{A}[\![a]\!]\sigma)/X], \quad \mathcal{C}[\![c_0; c_1]\!]\sigma = \mathcal{C}[\![c_1]\!](\mathcal{C}[\![c_0]\!]\sigma)$$

$$\frac{\mathcal{C}[\![b]\!]\sigma = \texttt{true} \quad \mathcal{C}[\![c_0]\!]\sigma = \sigma'}{\mathcal{C}[\![\texttt{if } b \texttt{ then } c_0 \texttt{ else } c_1]\!]\sigma = \sigma'}, \quad \frac{\mathcal{C}[\![b]\!]\sigma = \texttt{false} \quad \mathcal{C}[\![c_1]\!]\sigma = \sigma'}{\mathcal{C}[\![\texttt{if } b \texttt{ then } c_0 \texttt{ else } c_1]\!]\sigma = \sigma'}$$

$$\mathcal{C}[\![\texttt{while } b \texttt{ do } c]\!]\sigma = (fix(\Gamma))\sigma$$

donde $(fix(\Gamma))$ es el menor punto fijo de $\Gamma : \Sigma \to \Sigma$ definida por:

$$\Gamma(f)\sigma = \begin{cases} f(\mathcal{C}[\![c]\!]\sigma) & \text{si } \mathcal{B}[\![b]\!]\sigma = \texttt{true} \\ \sigma & \text{si } \mathcal{B}[\![b]\!]\sigma = \texttt{false} \end{cases}$$