Discrete-Time Fourier Transform Properties

	Sequence Domain	Frequency Domain
Linearity	$a_1 s_1(n) + a_2 s_2(n)$	$a_1 S_1\left(e^{j2\pi f}\right) + a_2 S_2\left(e^{j2\pi f}\right)$
Conjugate Symmetry	$s(n)$ real	$S\left(e^{j2\pi f}\right) = S\left(e^{-(j2\pi f)}\right)^*$
Even Symmetry	$s(n) = s(-n)$	$S\left(e^{j2\pi f}\right) = S\left(e^{-(j2\pi f)}\right)$
Odd Symmetry	$s(n) = -(s(-n))$	$S\left(e^{j2\pi f}\right) = -\left(S\left(e^{-(j2\pi f)}\right)\right)$
Time Delay	$s(n - n_0)$	$e^{-(j2\pi f n_0)} S\left(e^{j2\pi f}\right)$
Complex Modulation	$e^{j2\pi f_0 n} s(n)$	$S\left(e^{j2\pi(f-f_0)}\right)$
Amplitude Modulation	$s(n)\cos(2\pi f_0 n)$	$\frac{S\left(e^{j2\pi(f-f_0)}\right)+S\left(e^{j2\pi(f+f_0)}\right)}{2}$
	$s(n)\sin(2\pi f_0 n)$	$\frac{S\left(e^{j2\pi(f-f_0)}\right)-S\left(e^{j2\pi(f+f_0)}\right)}{2j}$
Multiplication by n	$ns(n)$	$\frac{1}{-(2j\pi)}\frac{d}{df}\left(S\left(e^{j2\pi f}\right)\right)$
Sum	$\sum_{n=-\infty}^{\infty}(s(n))$	$S\left(e^{j2\pi 0}\right)$
Value at Origin	$s(0)$	$\int_{-\left(\frac{1}{2}\right)}^{\frac{1}{2}} S\left(e^{j2\pi f}\right) df$
Parseval's Theorem	$\sum_{n=-\infty}^{\infty}\left(\left(\lvert s(n)\rvert\right)^2\right)$	$\int_{-\left(\frac{1}{2}\right)}^{\frac{1}{2}} \left(\lvert S\left(e^{j2\pi f}\right)\rvert\right)^2 df$

	Secuencia ($x[n]$)	**Transformada** ($X\left(e^{j\omega}\right)$)
1	$\delta[n]$	1
2	$\delta[n - n_o]$	$e^{-j\omega n_0}$
3	$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4	$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1 - a \cdot e^{-j\omega}}$
5	$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6	$(n+1)\cdot a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{\left(1 - a \cdot e^{-j\omega}\right)^2}$
7	$\frac{r^n \cdot \sin(\omega_p \cdot (n+1))}{\sin(\omega_p)} \cdot u[n] \quad (\lvert r\rvert < 1)$	$\frac{1}{1 - 2r \cdot \cos(\omega_p) \cdot e^{-j\omega} + r^2 \cdot e^{-j2\omega}}$
8	$\frac{\sin(\omega_c n)}{\pi n}$	$\begin{cases} 1, & \lvert\omega\rvert < \omega_c \\ 0, & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
9	$\begin{cases} 1, & 0 \le n \le M \\ 0, & \text{en otro caso} \end{cases}$	$\frac{\sin[\omega \cdot (M+1)/2]}{\sin(\omega/2)} \cdot e^{-j\omega\frac{M}{2}}$
10	$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11	$\cos(\omega_0 n + \phi)$	$\pi \cdot \sum_{k=-\infty}^{\infty}\left[e^{j\phi}\delta(\omega - \omega_0 + 2\pi k) + e^{-j\phi}\delta(\omega + \omega_0 + 2\pi k)\right]$

PROPIEDADES DE LA DFT

Property	Time Domain	Frequency Domain
Notation	$x(n), y(n)$	$X(k), Y(k)$
Periodicity	$x(n) = x(n+N)$	$X(k) = X(k+N)$
Linearity	$a_1 x_1(n) + a_2 x_2(n)$	$a_1 X_1(k) + a_2 X_2(k)$
Time Reversal	$x(N-n)$	$X(N-k)$
Circular Time Shift	$x((n-l))_N$	$X(k)e^{-j2\pi kl/N}$
Circular Frequency Shift	$x(n)e^{j2\pi nl/N}$	$X((k-l))_N$
Complex Conjugate	$x^*(n)$	$X^*(N-k)$
Circular Convolution	$x_1(n) \oplus x_2(n)$	$X_1(k)X_2(k)$
Circular Correlation	$x(n) \oplus y^*(-n)$	$X(k)Y^*(k)$
Multiplication	$x_1(n)x_2(n)$	$\frac{1}{N}X_1(k) \oplus X_2(k)$
Parseval's Theorem	$\sum_{n=0}^{N-1} x(n)y^*(n)$	$\frac{1}{N}\sum_{n=0}^{N-1} X(k)Y^*(k)$

N-Point Sequence	N-Point DFT
$x(n)$	$X(k)$
$x^*(n)$	$X^*(N-k)$
$x^*(N-n)$	$X^*(k)$
$x_R(n)$	$X_{ce}(k) = \frac{1}{2}[X(k) + X^*(N-k)]$
$jx_I(n)$	$X_{co}(k) = \frac{1}{2}[X(k) - X^*(N-k)]$
$x_{ce}(n) = \frac{1}{2}[x(n) + x^*(N-n)]$	$X_R(k)$
$x_{ce}(n) = \frac{1}{2}[x(n) + x^*(N-n)]$	$jX_I(k)$
Real Signals	
Any real signal $x(n)$	$X(k) = X^*(N-k)$ $X_R(k) = X_R(N-k)$ $X_I(k) = -X_I(N-k)$ $\lvert X(k)\rvert = \lvert X(N-k)\rvert$ $\angle X(k) = -\angle X(N-k)$
$x_{ce}(n) = \frac{1}{2}[x(n) + x(N-n)]$	$X_R(k)$
$x_{ce}(n) = \frac{1}{2}[x(n) - x^*(N-n)]$	$jX_I(k)$

TRANFORMADAS Z BÁSICAS

	Secuencia	Transformada	ROC
1	$\delta[n]$	1	$Todo\ z$
2	$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3	$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
4	$\delta[n-m]$	z^{-m}	$Todo\ z\ excepto\ 0\ (si\ m>0)$ $o\ \infty\ (si\ m<0)$
5	$a^n u[n]$	$\frac{1}{1-a\cdot z^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
6	$-a^n u[-n-1]$	$\frac{1}{1-a\cdot z^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
7	$n\,a^n u[n]$	$\frac{a\,z^{-1}}{(1-a\cdot z^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
8	$-n\,a^n u[-n-1]$	$\frac{a\,z^{-1}}{(1-a\cdot z^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
9	$[\cos\omega_o n]\cdot u[n]$	$\frac{1-[\cos\omega_o]z^{-1}}{1-[2\cdot\cos\omega_o]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10	$[sen\,\omega_o n]\cdot u[n]$	$\frac{[sen\,\omega_o]z^{-1}}{1-[2\cdot cos\,\omega_o]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
11	$[r^n\cos\omega_o n]\cdot u[n]$	$\frac{1-[r\cdot\cos\omega_o]z^{-1}}{1-[2\cdot r\cdot\cos\omega_o]z^{-1}+r^2\cdot z^{-2}}$	$\lvert z\rvert > r$
12	$[r^n sen\,\omega_o n]\cdot u[n]$	$\frac{[r\cdot sen\,\omega_o]z^{-1}}{1-[2\cdot r\cdot cos\,\omega_o]z^{-1}+r^2\cdot z^{-2}}$	$\lvert z\rvert > r$
13	$\begin{cases} a^n & 0\le n\le N-1 \\ 0 & en\ el\ resto \end{cases}$	$\frac{1-a^N\cdot z^{-N}}{1-a\cdot z^{-1}}$	$\lvert z\rvert > 0$

PROPIEDADES Z

PROPIEDAD	Secuencia	Transformada z	ROC
Linealidad	$a_1 x_1[n] + a_2 x_2[n]$	$a_1 X_1(z) + a_2 X_2(z)$	$\{R_{x_1} \cap R_{x_2}\} \subset ROC$
Desplazamiento temporal	$x[n - n_o]$	$z^{-n_o} X(z)$	$ROC = R_x$
Escalado en frecuencia	$z_o^n x[n]$	$X(z / z_o)$	$ROC = \lvert z_o \rvert \cdot R_x$
Reflexión temporal	$x^*[-n]$	$X^*(1 / z^*)$	$ROC = 1 / R_x$
Diferenciación de $X(z)$	$n \cdot x[n]$	$-z \cdot \frac{d}{dz} X(z)$	$ROC = R_x$
Convolución de secuencias	$x_1[n] * x_2[n]$	$X_1(z) \cdot X_2(z)$	$\{R_{x_1} \cap R_{x_2}\} \subset ROC$
Teorema del valor inicial	$x[0]$	$\lim_{z \to \infty} X(z)$	