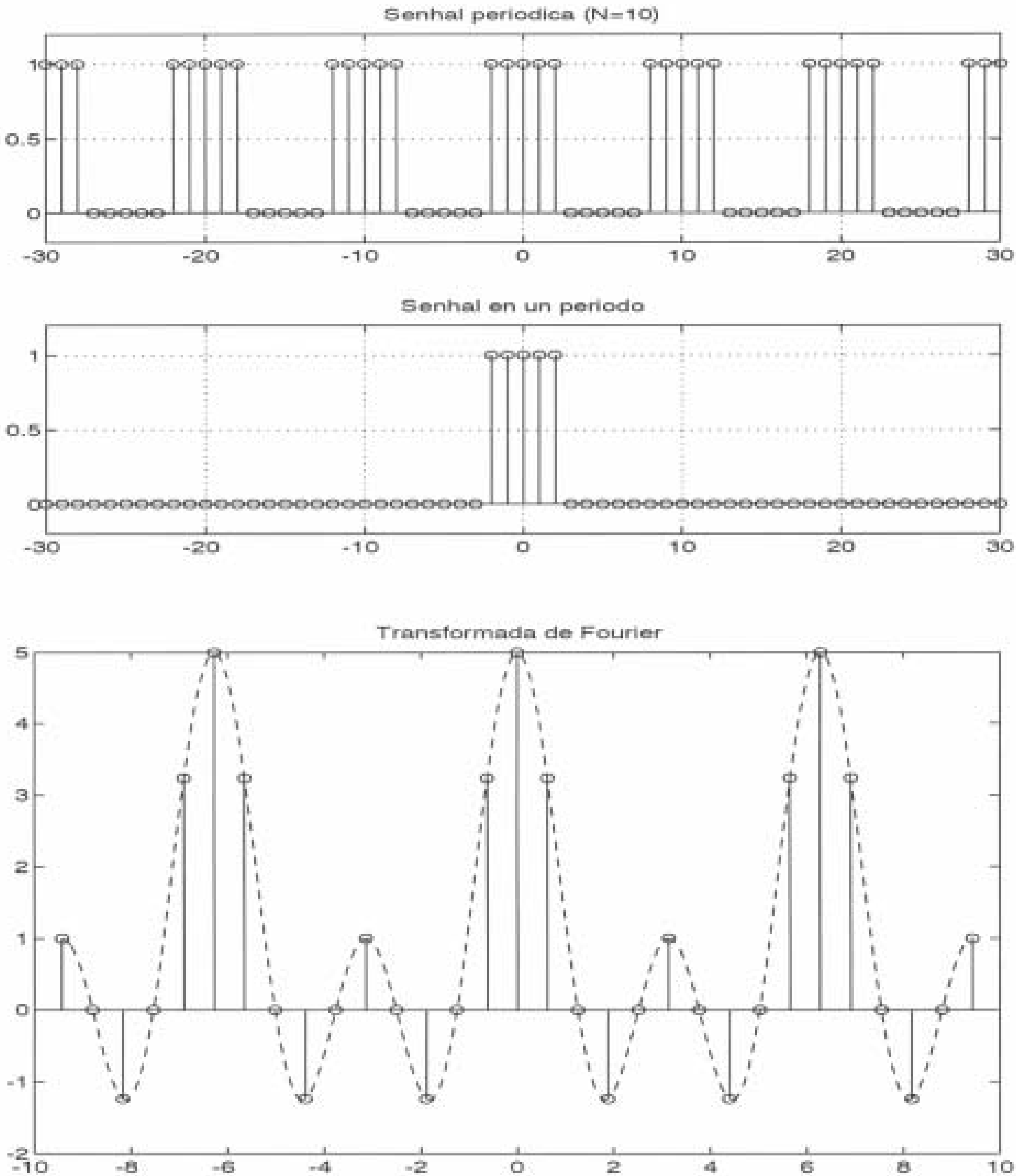
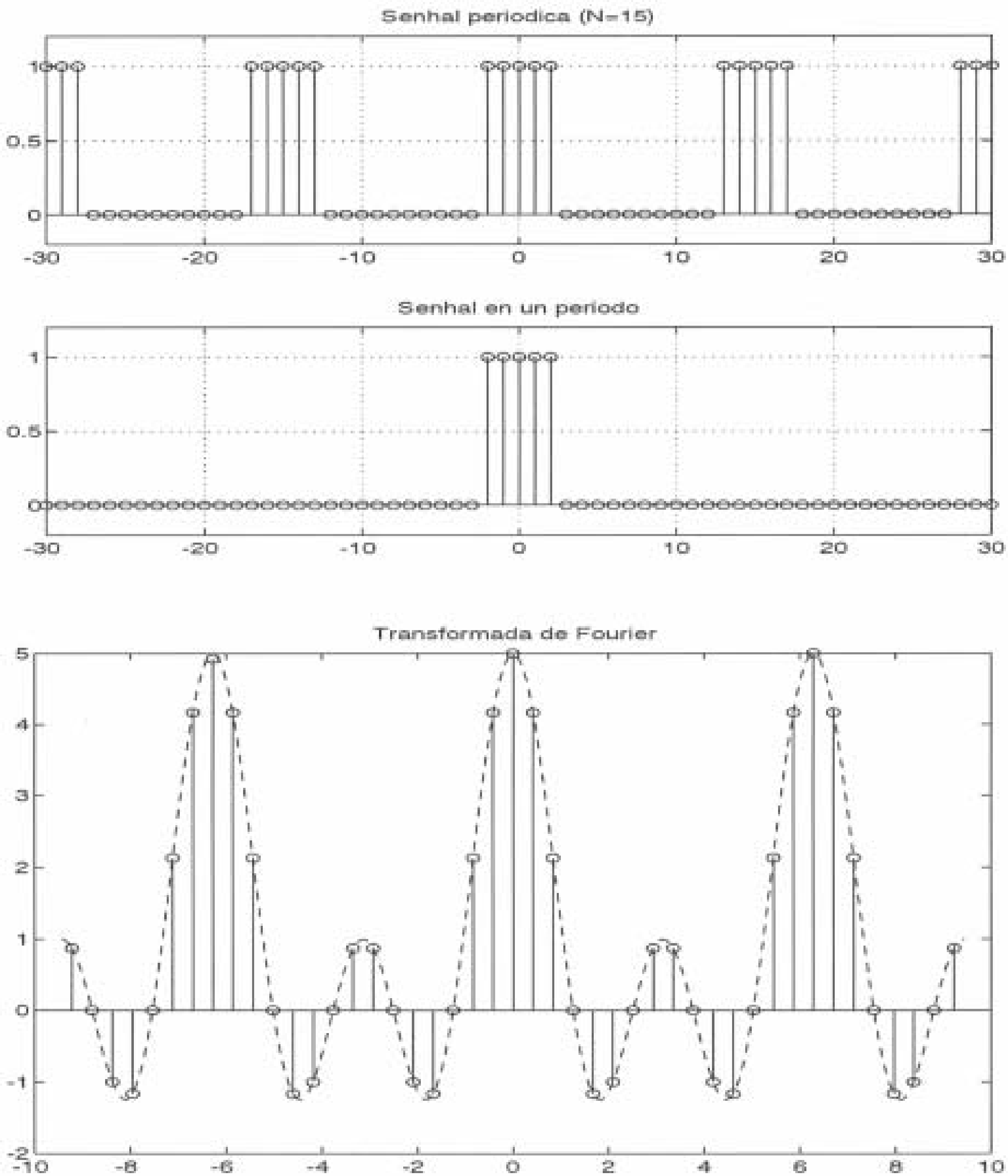
Tratamiento Digital de la Señal

Tema 5

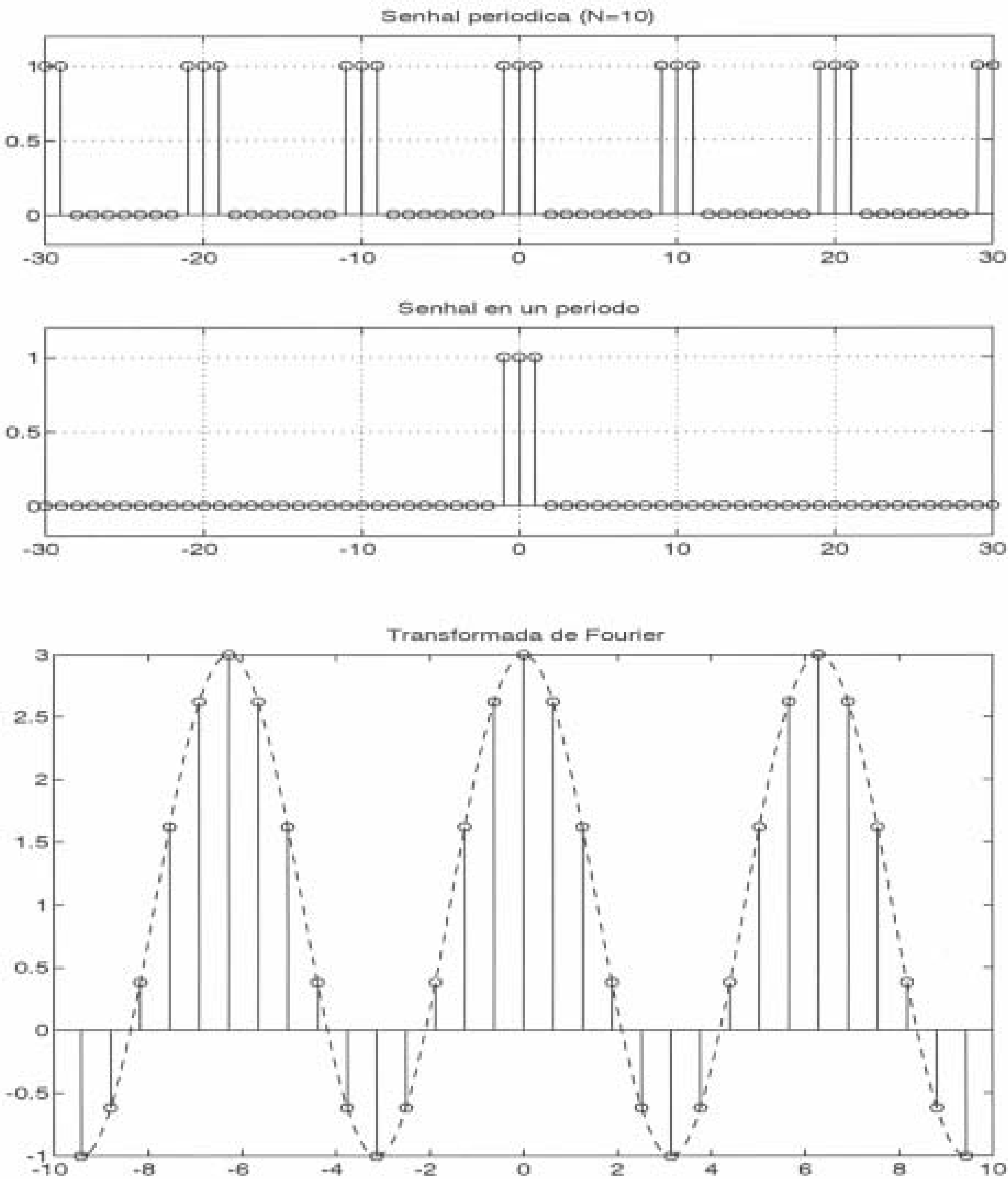
Transformada de Fourier de una secuencia periódica



Transformada de Fourier de una secuencia periódica



Transformada de Fourier de una secuencia periódica

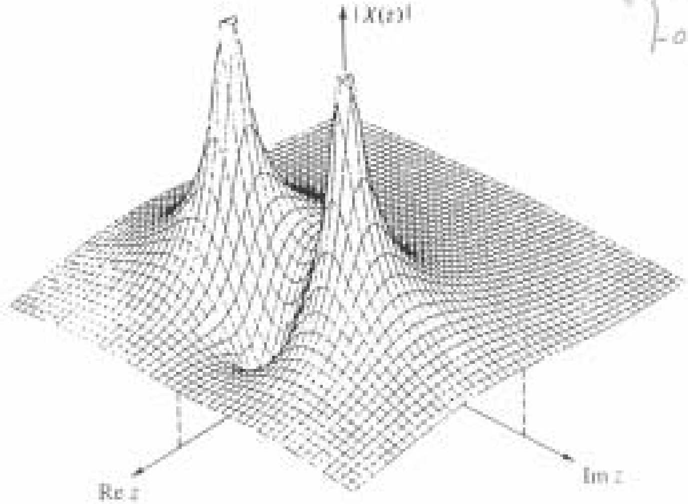


$$X(z) = \frac{z^{-1} - z^{-2}}{1 + 1.2732z^{-1} + 0.81z^{-2}}$$

$\rightarrow z = 1$

$\rightarrow z = -0.9 \, e^{\pm j \frac{\pi}{4}}$

$= \begin{cases} 0.636 + 0.636 j \\ -0.636 + 0.636 j \end{cases}$



(a)

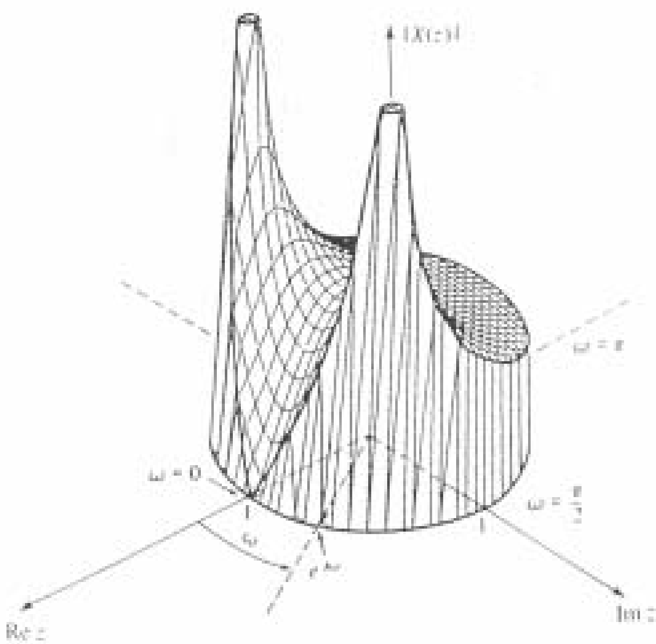


Figure 3.10 Graph of $|X(z)|$ for the z-transform in (3.3.3). [Reproduced with permission from *Introduction to Systems Analysis*, by T. H. Glisson, © 1985 by McGraw-Hill Book Company.]

As an example of a system function with both poles and zeros, consider

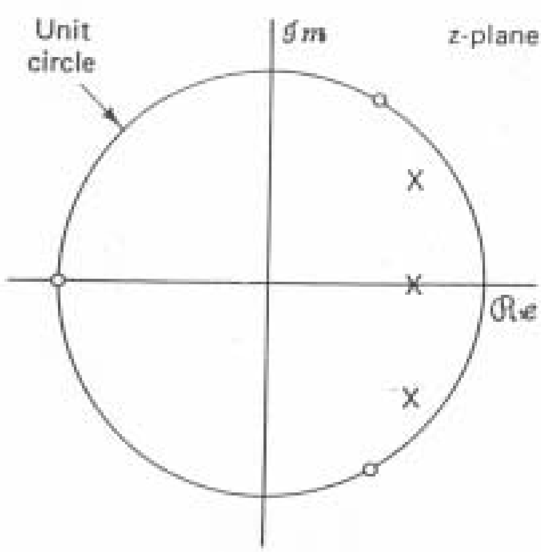
$$H(z) = \frac{0.05634(1 + z^{-1})(1 - 1.0166z^{-1} + z^{-2})}{(1 - 0.683z^{-1})(1 - 1.4461z^{-1} + 0.7957z^{-2})}.$$

The zeros of this system function are at

Radius	Angle
1	π rad
1	$\pm$ 1.0376 rad (59.45°)

The poles are at

Radius	Angle
0.683	0
0.892	$\pm$ 0.6257 rad (35.85°)



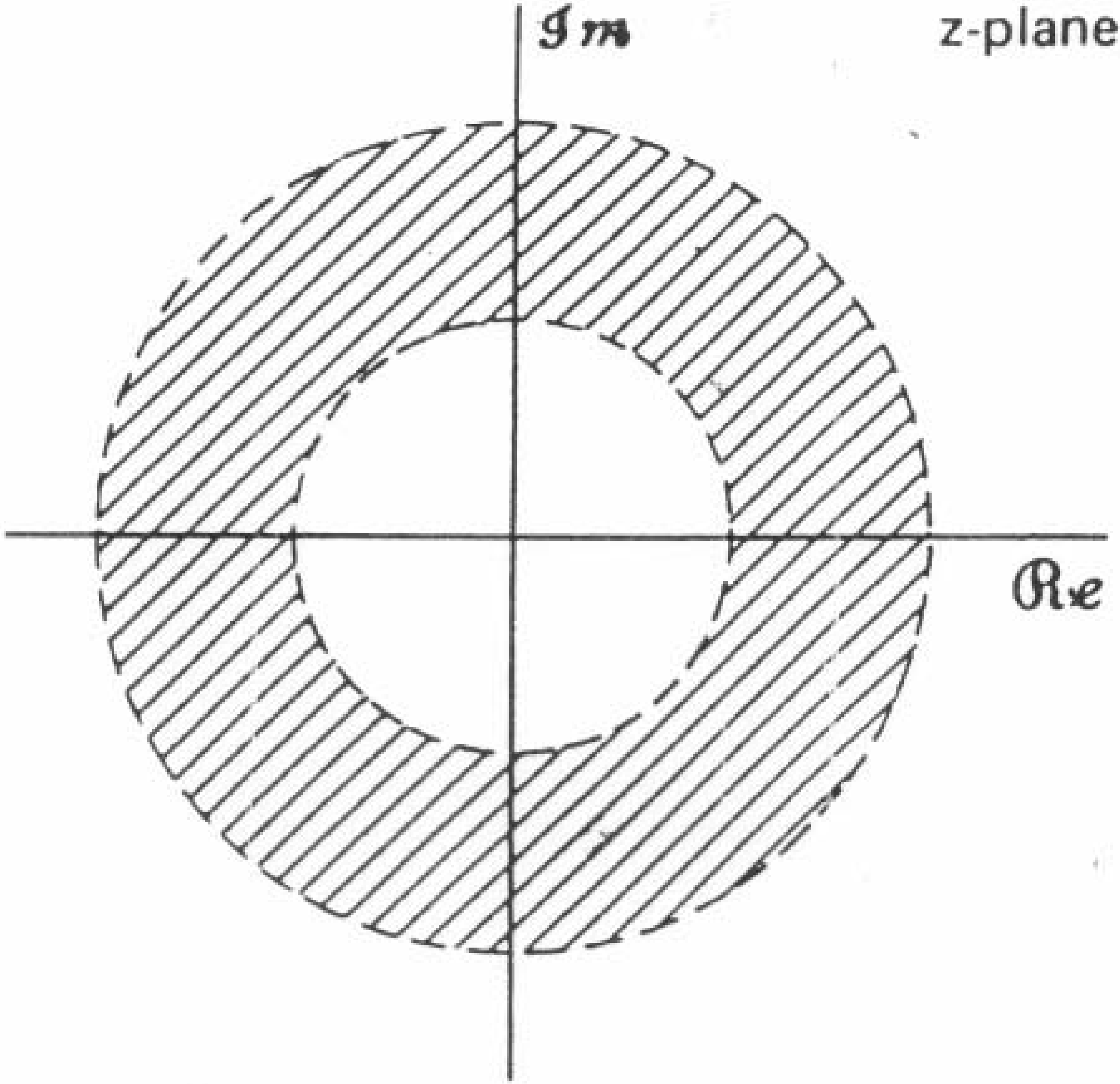
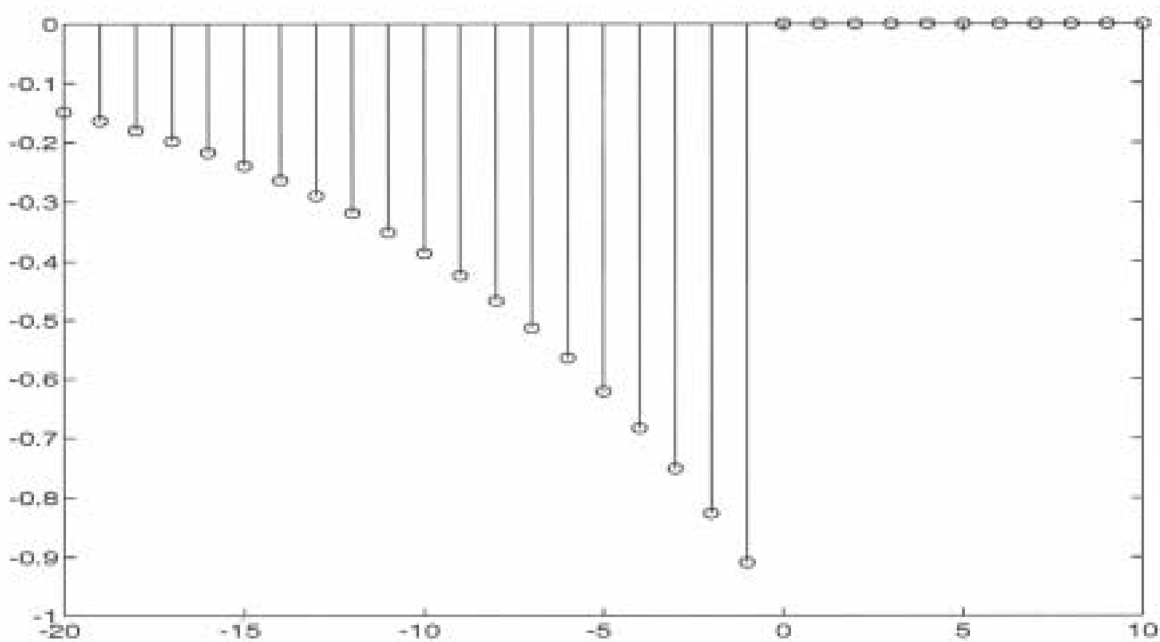
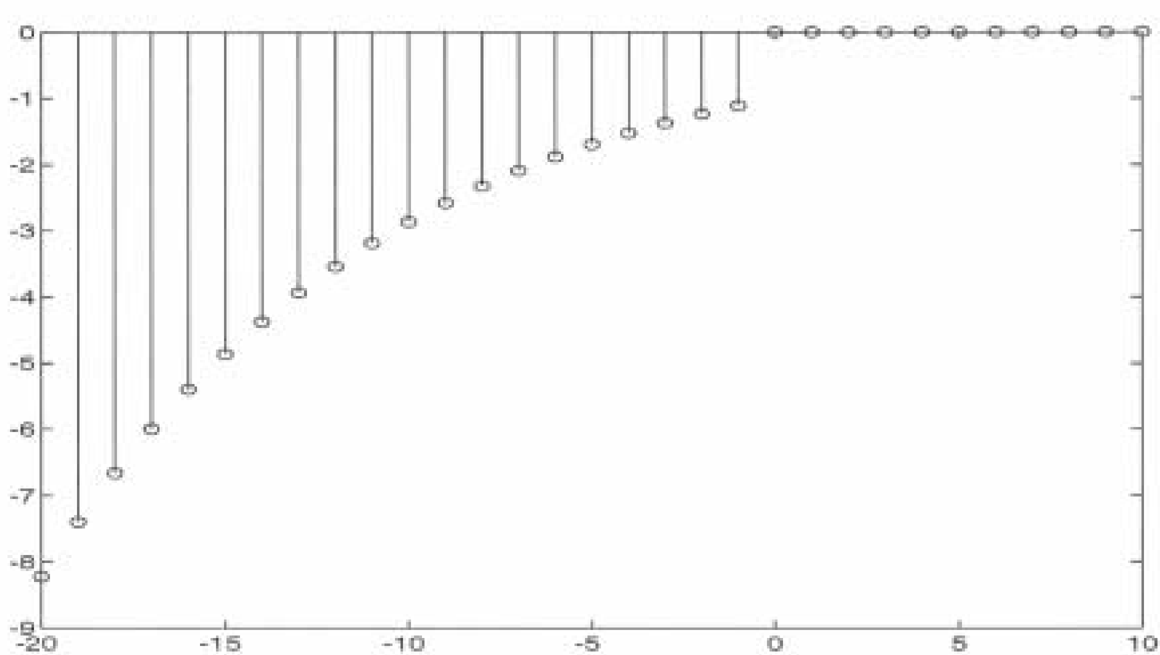


Figure 4.2 The region of convergence (ROC) as a ring in the z-plane. For specific cases, the inner boundary can extend inward to the origin, and the ROC becomes a disc. For other cases, the outer boundary can extend outward to infinity.

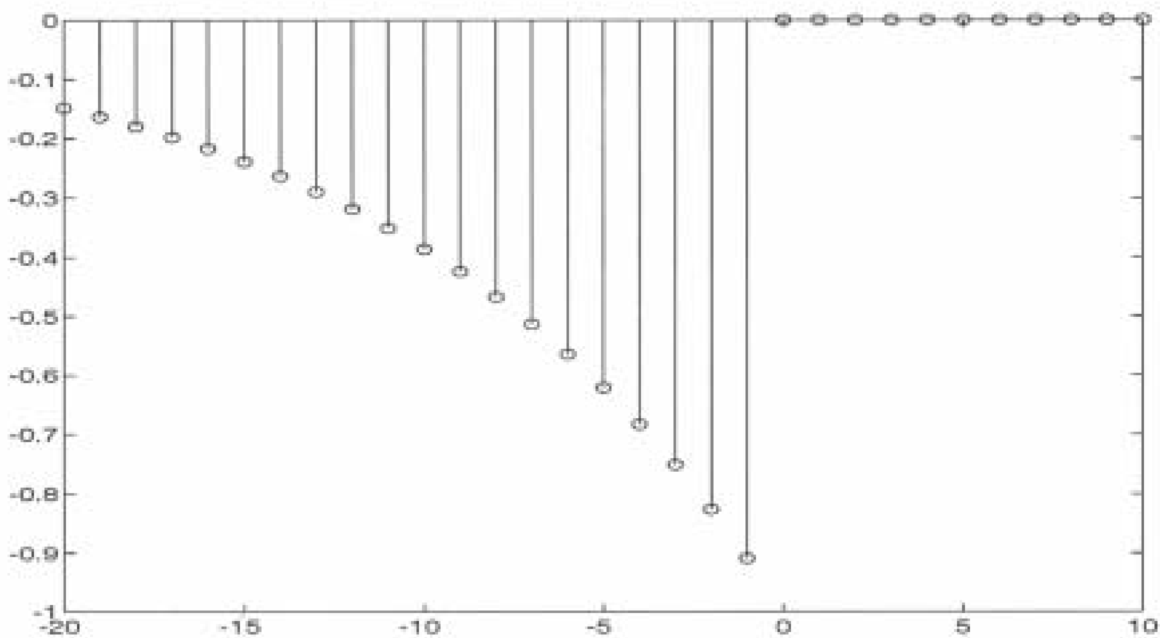
$$x(n) = -(1.1)^n \, u(-n-1)$$



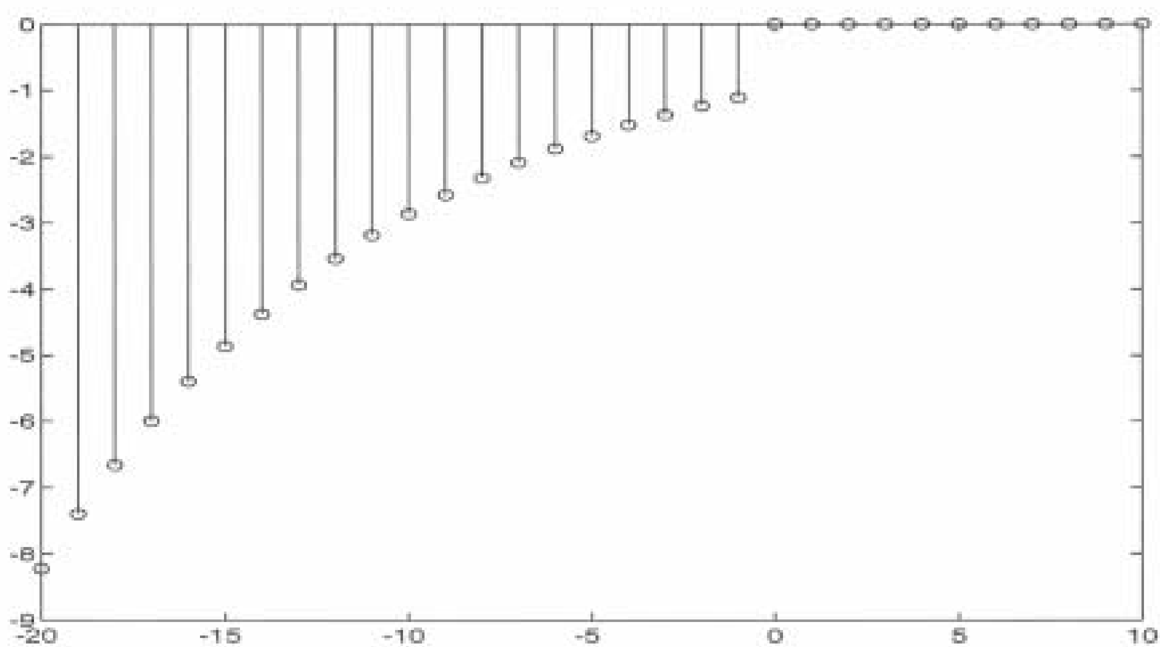
$$x(n) = -(0.9)^n \, u(-n-1)$$



$$x(n) = -(1.1)^n \, u(-n-1)$$



$$x(n) = -(0.9)^n \, u(-n-1)$$



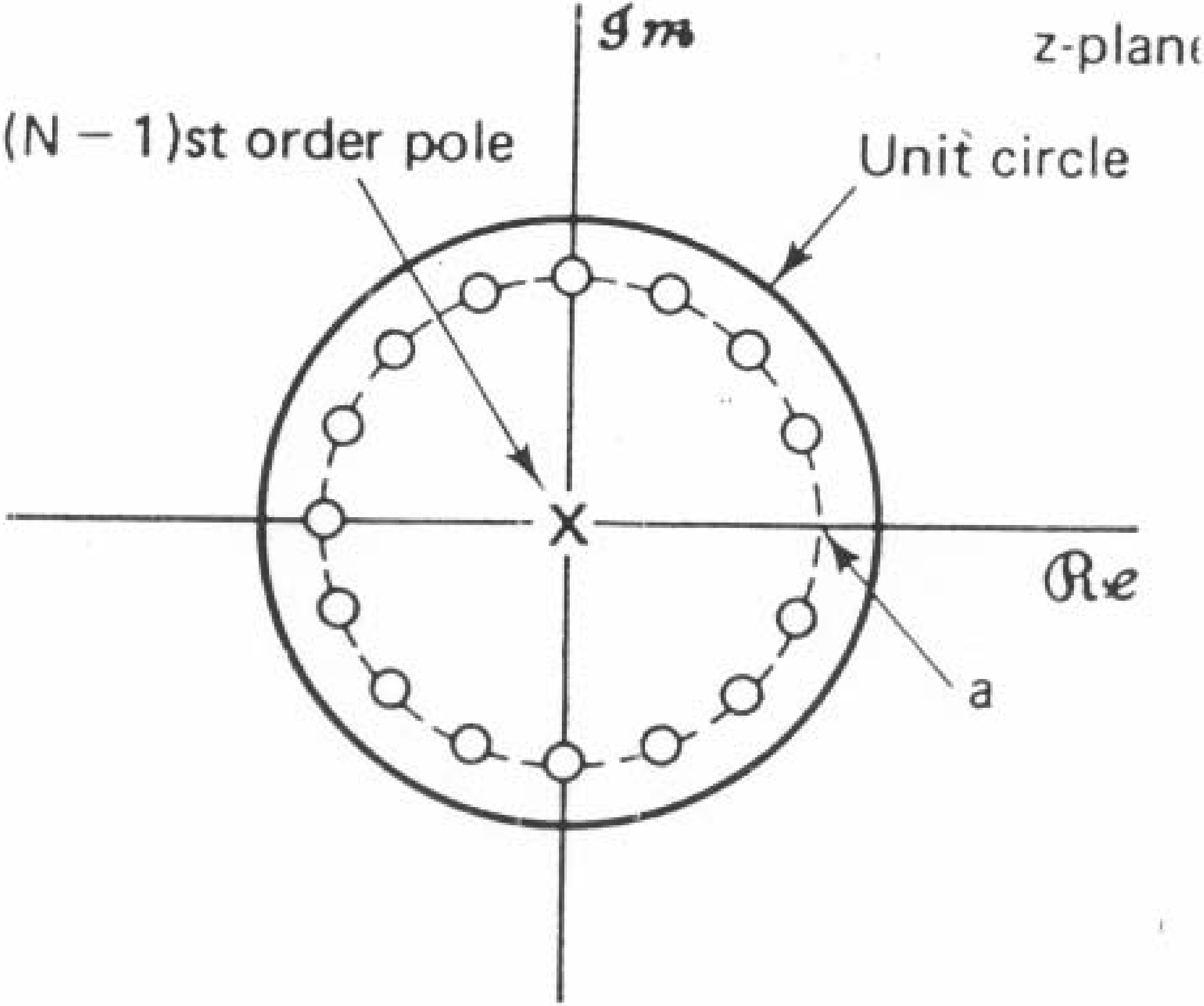


Figure 4.7 Pole-zero plot for Example 4.6 with $N = 16$ and a real such that $0 < a < 1$. The region of convergence for this example consists of all values of z except $z = 0$.

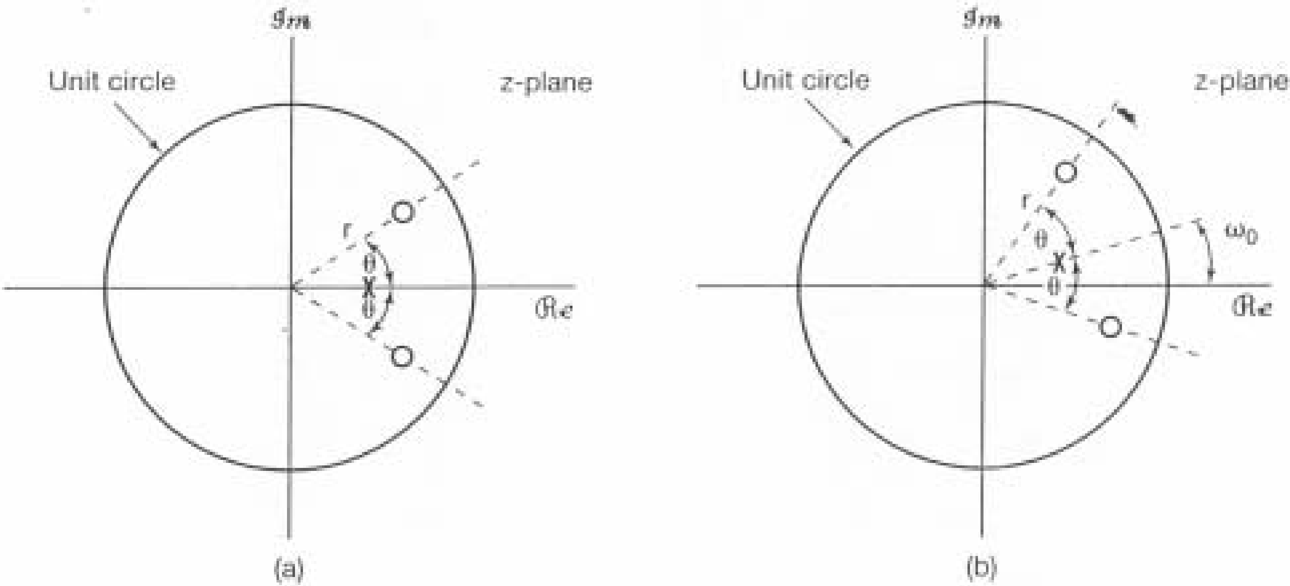


Figure 10.15 Effect on the pole-zero plot of time-domain multiplication by a complex exponential sequence $e^{j\omega_0 n}$: (a) pole-zero pattern for the z-transform for a signal $x[n]$; (b) pole-zero pattern for the z-transform of $x[n]e^{j\omega_0 n}$.

Section	Property	Signal	z-Transform	ROC
		$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R R_1 R_2
10.5.1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
10.5.2	Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R, except for the possible addition or deletion of the origin
10.5.3	Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
		$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
		$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $\|a\|R$ = the set of points $\{\|a\|z\}$ for z in R)
10.5.4	Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1}, where z is in R)
10.5.5	Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$, where z is in R)
10.5.6	Conjugation	$x^*[n]$	$X^*(z^*)$	R
10.5.7	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
10.5.7	First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
10.5.7	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
10.5.8	Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R
10.5.9		Initial Value Theorem If $x[n] = 0$ for $n < 0$, then $x[0] = \lim_{z \to \infty} X(z)$		