

1. Calcular y simplificar: $wp("b[i] := b[j]; b[j] := b[i]", b[b[i]] = b[j])$

$$wp("b[i] := b[j]; b[j] := b[i]", b[b[i]] = b[j]) \equiv wp("b[i] := b[j]", wp("b[j] = b[i]", b[b[i]] = b[j]))$$

Primero calcularemos:

$$wp("b[j] := b[i]", b[b[i]] = b[j]) = \text{enrango}(b, i) \text{ cand } \text{dominio}(b[i]) \text{ cand } \text{enrango}(b, j) \text{ cand } (b[b[i]] = b[j])_{(b; j: b[i])}^b$$

Desarrollando el último término obtenemos:

$$\begin{aligned} & (b[b[i]] = b[j])_{(b; j: b[i])}^b \\ \equiv & (b; j : b[i])[b[i]] = (b; j : b[i])[j] \\ \equiv & (b; j : b[i])[b[i]] = b[i] \\ \equiv & (j = i \wedge (b; j : b[i])[b[i]] = b[i]) \vee (j \neq i \wedge (b; j : b[i])[b[i]] = b[i]) \end{aligned}$$

Sacando factor común el segundo término de ambas conjunciones tenemos:

$$\begin{aligned} \equiv & (j = i \vee j \neq i) \wedge ((b; j : b[i])[b[i]] = b[i]) \\ \equiv & (b; j : b[i])[b[i]] = b[i] \\ \equiv & (j = b[i] \wedge b[i] = b[i]) \vee (j \neq b[i] \wedge b[b[i]] = b[i]) \\ \equiv & j = b[i] \vee (j \neq b[i] \wedge b[b[i]] = b[i]) \\ \equiv & j = b[i] \vee b[b[i]] = b[i] \end{aligned}$$

A continuación calculamos:

$$\begin{aligned} & wp("b[i] := b[j]", \text{enrango}(b, i) \text{ cand } \text{dominio}(b[i]) \text{ cand } \text{enrango}(b, j) \text{ cand } (j = b[i] \vee b[b[i]] = b[i])) \\ \equiv & \text{enrango}(b, j) \text{ cand } \text{dominio}(b[j]) \text{ cand } \text{enrango}(b, i) \text{ cand } \text{enrango}((b; i : b[j]), i) \text{ cand } \\ & \text{dominio}(b[i]) \text{ cand } \text{enrango}((b; i : b[j]), j) \text{ cand } (j = b[i] \vee b[b[i]] = b[i])_{(b; i: b[j])}^b \\ \equiv & \text{enrango}(b, j) \text{ cand } \text{dominio}(b[j]) \text{ cand } \text{enrango}(b, i) \text{ cand } \text{enrango}(b, i) \text{ cand } \\ & \text{dominio}(b[i]) \text{ cand } \text{enrango}(b, i) \text{ cand } \text{enrango}(b, j) \text{ cand } (j = b[i] \vee b[b[i]] = b[i])_{(b; i: b[j])}^b \\ \equiv & \text{enrango}(b, j) \text{ cand } \text{dominio}(b[j]) \text{ cand } \text{enrango}(b, i) \text{ cand } \text{dominio}(b[i]) \text{ cand } (j = b[i] \vee b[b[i]] = b[i])_{(b; i: b[j])}^b \end{aligned}$$

Desarrollando el último término obtenemos:

$$\begin{aligned}
& (j = b[i] \vee b[b[i]] = b[i])_{(b; i: b[j])}^b \\
& \equiv (j = (b; i : b[j])[i] \vee (b; i : b[j])[(b; i : b[j])[i]] = (b; i : b[j])[i]) \\
& \equiv (j = b[j] \vee (b; i : b[j])[b[j]] = b[j]) \\
& \equiv (j = b[j] \vee (i = b[j] \wedge b[j] = b[j]) \vee (i \neq b[j] \wedge b[b[j]] = b[j])) \\
& \equiv (j = b[j] \vee i = b[j] \vee (i \neq b[j] \wedge b[b[j]] = b[j])) \\
& \equiv (j = b[j] \vee i = b[j] \vee b[b[j]] = b[j])
\end{aligned}$$

Con lo que nos queda como resultado final:

$$\begin{aligned}
& \overline{\text{enrango}(b, j) \text{ cand } \text{dominio}(b[j]) \text{ cand } \text{enrango}(b, i) \text{ cand } \text{dominio}(b[i])} \\
& \text{cand } (j = b[j] \vee i = b[j] \vee b[b[j]] = b[j])
\end{aligned}$$
