

# CÁLCULO

Boletín P. Cálculo de primitivas

1.  $I = \int (x^2 + 1) \ln x \, dx$  *Solución:*  $I = \frac{x}{3}(x^2 + 3) \ln x - \frac{x}{9}(x^2 + 9) + C$
2.  $I = \int (x^2 + x + 3) \cos x \, dx$  *Solución:*  $I = (x^2 + x + 1) \sin x + (2x + 1) \cos x + C$
3.  $I = \int 3^x (x^2 + 1) \, dx$  *Solución:*  $I = \frac{3^x}{(\ln 3)^3} [(x^2 + 1)(\ln 3)^2 - 2x \ln 3 + 2] + C$
4.  $I = \int e^{ax} \sin bx \, dx$  *Solución:*  $I = \frac{a \sin bx - b \cos bx}{a^2 + b^2} e^{ax} + C$
5.  $I = \int x e^{ax} \sin bx \, dx$  *Solución:*  $I = \frac{[(ax-1)(a^2+1)+2] \sin x + [(1-x)(a^2+1)-(a-1)^2] \cos x}{(a^2+1)^2} e^{ax} + C$
6.  $I = \int \frac{\ln x}{x^2} \, dx$  *Solución:*  $I = -\frac{1}{x} (1 + \ln x) + C$
7.  $I = \int (\arcsin x)^2 \, dx$  *Solución:*  $I = [(\arcsin x)^2 - 2] x + 2\sqrt{1-x^2} \arcsin x + C$
8.  $I = \int x^3 \cos x^2 \, dx$  *Solución:*  $I = \frac{1}{2} (x^2 \sin x^2 + \cos x^2) + C$
9.  $I = \int x \arctan x \, dx$  *Solución:*  $I = \frac{1}{2} [(x^2 + 1) \arctan x - x] + C$
10.  $I = \int \frac{\ln x}{x} \, dx$  *Solución:*  $I = \frac{1}{2} (\ln x)^2 + C$
11.  $I = \int x^2 \sin^2 x \, dx$  *Solución:*  $I = \frac{1}{24} [4x^3 - 6x \cos 2x + 3(1 - 2x^2) \sin 2x] + C$
12.  $I = \int \frac{x}{(1+x^2)^2} \ln x \, dx$  *Solución:*  $I = \frac{x^2}{2(1+x^2)} \ln x - \frac{1}{4} \ln(1+x^2) + C$
13.  $I = \int \frac{x}{(1+x^2)^2} \arctan x \, dx$  *Solución:*  $I = \frac{x}{4(1+x^2)} + \frac{x^2-1}{4(1+x^2)} \arctan x + C$
14.  $I = \int \ln(x^2 + 4x + 5) \, dx$  *Solución:*  $I = (x + 2) \ln(x^2 + 4x + 5) - 2x + 2 \arctan(x + 2) + C$
15.  $I = \int (1-x) \ln(1+x) \, dx$  *Solución:*  $I = \frac{3+2x-x^2}{2} \ln(1+x) + \frac{x^2-6x}{4} + C$
16.  $I = \int x \arcsin x \, dx$  *Solución:*  $I = -\frac{1}{4} (\arcsin x) (1 - 2x^2) + \frac{1}{4} x \sqrt{1-x^2} + C$
17.  $I = \int \frac{x^5 + 4x^3 + 7x + 4}{x^2 + 1} \, dx$  *Solución:*  $I = \frac{x^2}{4} (x^2 + 6) + 2 \ln(x^2 + 1) + 4 \arctan x + C$

18.  $I = \int \frac{x^2 + 3x + 4}{(x-1)(x+1)^2} dx$  *Solución:*  $I = 2 \ln|x-1| - \ln|x+1| + \frac{1}{x+1} + C$
19.  $I = \int \frac{3x + 5}{x^2 - x + 4} dx$  *Solución:*  $I = \frac{3}{2} \ln|x^2 - x + 4| + \frac{13}{\sqrt{15}} \arctan \frac{2x-1}{\sqrt{15}} + C$
20.  $I = \int \frac{x^3}{x^4 + x^2 + 1} dx$  *Solución:*  $I = \frac{1}{4} \ln(x^4 + x^2 + 1) - \frac{\sqrt{3}}{6} \arctan \frac{2x^2+1}{\sqrt{3}} + C$
21.  $I = \int \frac{x^2 - x + 3}{(x+2)^2} dx$  *Solución:*  $I = x - \frac{9}{x+2} - 5 \ln|x+2| + C$
22.  $I = \int \frac{2x^2 - x - 2}{(x-3)(x^2 + x + 1)} dx$  *Solución:*  $I = \ln|x-3| + \frac{1}{2} \ln|x^2 + x + 1| + \frac{\sqrt{3}}{3} \arctan \frac{2x+1}{\sqrt{3}} + C$
23.  $I = \int \frac{x^4}{x^4 - 1} dx$  *Solución:*  $I = x + \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \arctan x + C$
24.  $I = \int \frac{5x - 1}{x^3 + x^2 - x - 1} dx$  *Solución:*  $I = \ln \left| \frac{x-1}{x+1} \right| - \frac{3}{1+x} + C$
25.  $I = \int \frac{x^5}{1 + x^4} dx$  *Solución:*  $I = \frac{1}{2}(x^2 - \arctan x^2) + C$
26.  $I = \int \frac{(2x^3 - 1)x^2}{(1 + x^3)^3} dx$  *Solución:*  $I = -\frac{4x^3+1}{6(1+x^3)^2}$
27.  $I = \int \frac{4x^2 - 3x + 12}{(x-3)^2(x^2 + x + 1)} dx$  *Solución:*  $I = \frac{2}{\sqrt{3}} \arctan \frac{2x+1}{\sqrt{3}} - \frac{3}{x-3} + C$
28.  $I = \int \frac{\sqrt{x} - 1}{6(x^{1/3} + 1)} dx$   
*Solución:*  $I = \frac{x^{7/6}}{7} - \frac{x^{5/6}}{5} - \frac{x^{4/6}}{4} + \frac{x^{3/6}}{3} + \frac{x^{2/6}}{2} - x^{1/6} - \frac{1}{2} \ln(1 + x^{1/3}) + \arctan x^{1/6} + C$
29.  $I = \int \sqrt{\frac{3-x}{2+x}} dx$  *Solución:*  $I = (2+x)\sqrt{\frac{3-x}{2+x}} - 5 \arctan \sqrt{\frac{3-x}{2+x}} + C$
30.  $I = \int \sqrt{\frac{3-x}{3+x}} dx$  *Solución:*  $I = 3 \arcsin \frac{x}{3} + \sqrt{9-x^2} + C$
31.  $I = \int x^{-3/2}(1+x^{1/4})^2 dx$  *Solución:*  $I = -\frac{2}{\sqrt{x}} - \frac{8}{x^{1/4}} + \ln|x| + C$
32.  $I = \int \frac{x^3}{\sqrt{1+x^2}} dx$  *Solución:*  $I = \frac{1}{3}(x^2 - 2)\sqrt{1+x^2} + C$
33.  $I = \int \frac{x^2}{\sqrt{1-x^2}} dx$  *Solución:*  $I = -\frac{1}{2} \left[ \arccos x + x\sqrt{1-x^2} \right] + C$
34.  $I = \int \sin^5 x dx$  *Solución:*  $I = -\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$
35.  $I = \int \frac{dx}{\cos^3 x}$  *Solución:*  $I = \frac{1}{4} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + \frac{\sin x}{2 \cos^2 x} + C$
36.  $I = \int \tan^3 x dx$  *Solución:*  $I = \frac{1}{2} \tan^2 x + \ln|\cos x| + C$

$$37. I = \int \frac{dx}{1 + \cos^2 x} \quad \text{Solución: } I = \frac{\sqrt{2}}{2} \arctan \frac{\tan x}{\sqrt{2}} + C$$

$$38. I = \int \frac{dx}{1 + \sin x \cos x} \quad \text{Solución: } I = \frac{2}{\sqrt{3}} \arctan \frac{1+2\tan x}{\sqrt{3}} + C$$

$$39. I = \int \frac{dx}{1 + \sin x + \cos x} \quad \text{Solución: } I = \ln \left| 1 + \tan \frac{x}{2} \right| + C$$

$$40. I = \int \frac{dx}{5 + 3 \cos x} \quad \text{Solución: } I = \frac{1}{2} \arctan \left[ \frac{1}{2} \tan \frac{x}{2} \right] + C$$

$$41. I = \int \sin^5 x \cos^4 x dx \quad \text{Solución: } I = \frac{2}{7} \cos^7 x - \frac{1}{5} \cos^5 x - \frac{1}{9} \cos^9 x + C$$

$$42. I = \int \frac{dx}{a + b \sin x}, \quad a > b > 0 \quad \text{Solución: } I = \frac{2}{\sqrt{a^2 - b^2}} \arctan \frac{b + a \tan(x/2)}{\sqrt{a^2 - b^2}} + C$$

$$43. I = \int \tan^4 x dx \quad \text{Solución: } I = \frac{1}{3} \tan^3 x - \tan x + x + C$$

$$44. I = \int \frac{\sin^2 x}{1 + \cos^2 x} dx \quad \text{Solución: } I = \sqrt{2} \arctan \frac{\tan x}{\sqrt{2}} - x + C$$

$$45. I = \int \frac{dx}{\sin x - \tan x} \quad \text{Solución: } I = \frac{1}{4 \tan^2(x/2)} + \frac{1}{2} \ln |\tan(x/2)| + C$$

$$46. I = \int \frac{\sin^2 x}{\sin x \cos x + 2 \cos^4 x} dx \quad \text{Solución: } I = \frac{1}{4} \ln |1 + \tan x| + \frac{3}{8} \ln |\tan^2 x - \tan x + 2| - \frac{1}{4\sqrt{7}} \arctan \frac{2 \tan x - 1}{\sqrt{7}} + C$$

$$47. I = \int \frac{1}{4} \frac{\sin^{1/2} x + \sin^{1/4} x}{\sin^{1/4} x - 1} \cos x dx \quad \text{Solución: } I = \frac{1}{5} \sin^{5/4} x + \frac{1}{2} \sin x + \frac{2}{3} \sin^{3/4} x + \sin^{1/2} x + 2 \sin^{1/4} x + 2 \ln |\sin^{1/4} x - 1| + C$$

$$48. I = \int \frac{\cos^3 x}{1 + \sin^2 x} dx \quad \text{Solución: } I = -t + 2 \arctan t + C = 2 \arctan(\sin x) - \sin x + C$$

$$49. I = \int \frac{\sin^2 x \tan x}{4 + \cos^2 x} dx \quad \text{Solución: } I = \frac{5}{8} \ln \left| 1 + \frac{4}{\cos^2 x} \right| + \ln |\cos x| + C$$